St. Lawrence Seaway

Learning and Evaluation Situation

MTH-5173-2

Geometric Representation in a Fundamental Context 2

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Summary Situation on Vectors

(Learning Help Evaluation)

Adult Learners' Workbook

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Presentation of the situation

The St. Lawrence Seaway is a major transportation route for freight as well as people. In addition to being a transportation link, the Seaway is also a popular tourist attraction. However, because of its variable currents, numerous tributaries and narrow and shallow sections, the Seaway requires a great deal of skill to navigate. In this learning ~~and~~ and evaluation situation, you will present three particularly complex navigation situations in the Sorel-Tracy and Lac Saint-Pierre region using vectors. You will also be required to complete your presentation by demonstrating a vector-related concept.

PEDAGOGICAL INTENTION

Interpret reality using vectors;

Apply the acquired knowledge to real-life situations;

Evaluate the level of skill required to solve complex problems. (Competency 1 is particularly sought.)

INTEGRATIVE PROCESS:

Using vectors to generalize geometry principles

CROSS-CURRICULAR COMPETENCIES

- *Uses information and communications technologies.* You will be able to use GeoGebra software to review vector operations and to illustrate the presented situations.

SUBJECT-SPECIFIC COMPETENCIES

- *Uses strategies to solve situational problems.* You must identify the elements you will require and represent the situation. The representation may be provided to you, if necessary.
- *Uses mathematical reasoning.* You must solve problems by correctly applying the skills acquired in this course.
- *Communicates by using mathematical language.* You must observe vector notation and representation conventions.

Skills

Did you know that the Lac-Saint-Pierre archipelago, near Sorel, is home to the largest known heronry in the world?

Vectors

- Resultant and projection
- Operations on vectors
  - Adding and subtracting vectors
  - Multiplying a vector by a scalar
  - Scalar product of two vectors
  - Properties of scalar product of two vectors
  - Linear combination
  - Properties of vectors
- Determining the coordinates of a point of division
- Principles P22 to P26

Expected outcome

At the end of this learning situation, you will be able to apply your knowledge of vectors to solve real-life and situations and demonstrations.

Resources

GeoGebra Book

http://monurl.ca/mat5173

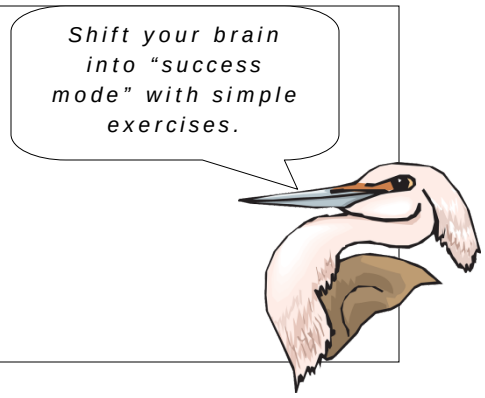
Allô prof

http://monurl.ca/vecteurs

Knowledge evaluation

Demonstrate properties using GeoGebra.

1. Trace the following vectors and calculate their norm and orientation angle.
   a. $\vec{u} = (3, -2)$: norm: _____ Angle: _____
   b. $\vec{v} = (4; 2{,}5)$: norm: _____ Angle: _____
   c. $\vec{w} = (-1{,}6)$: norm: _____ Angle: _____



2. Using the GeoGebra activities, associate the vector property with each demonstration.

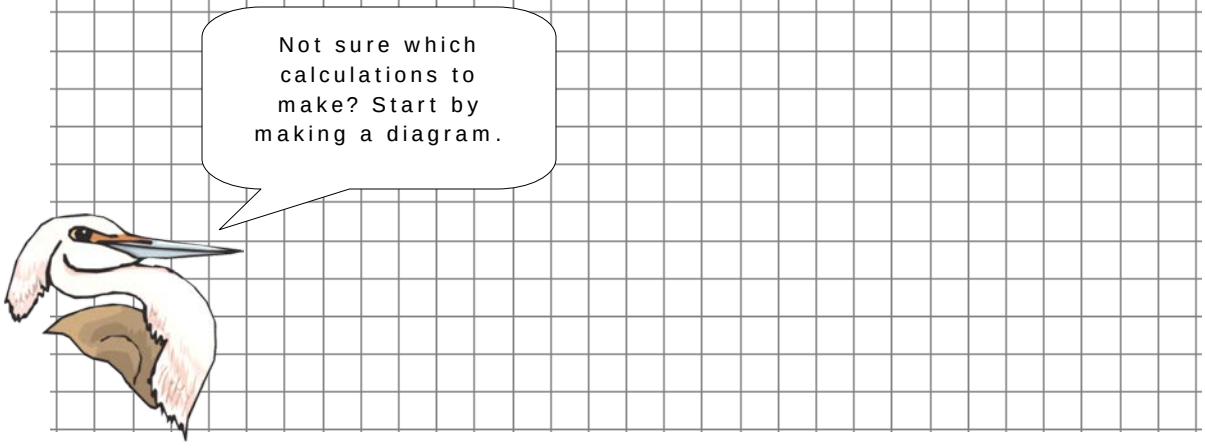
   a. Demonstration 1: _____

   b. Demonstration 2: _____

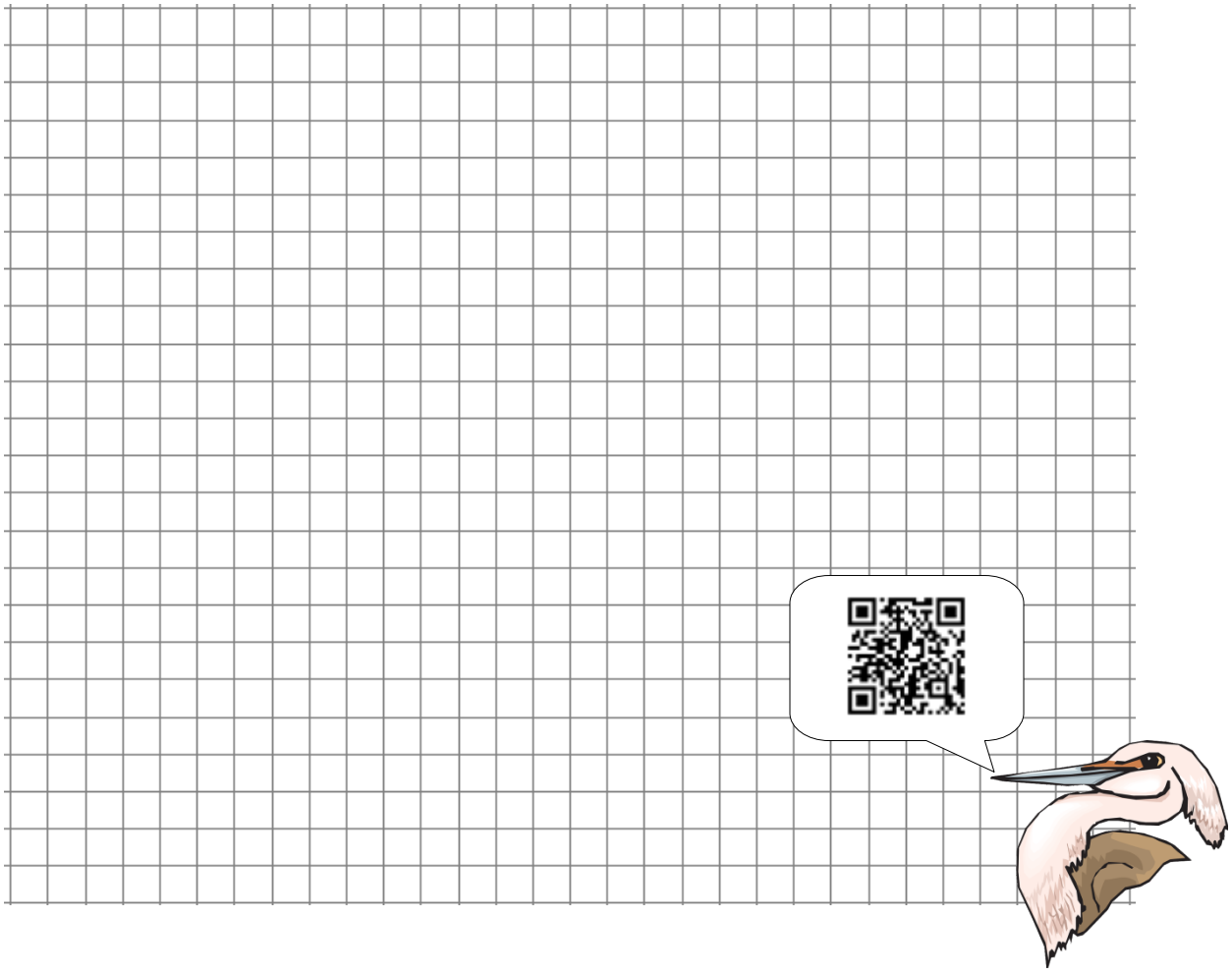
   c. Demonstration 3: _____

   d. Demonstration 4: _____

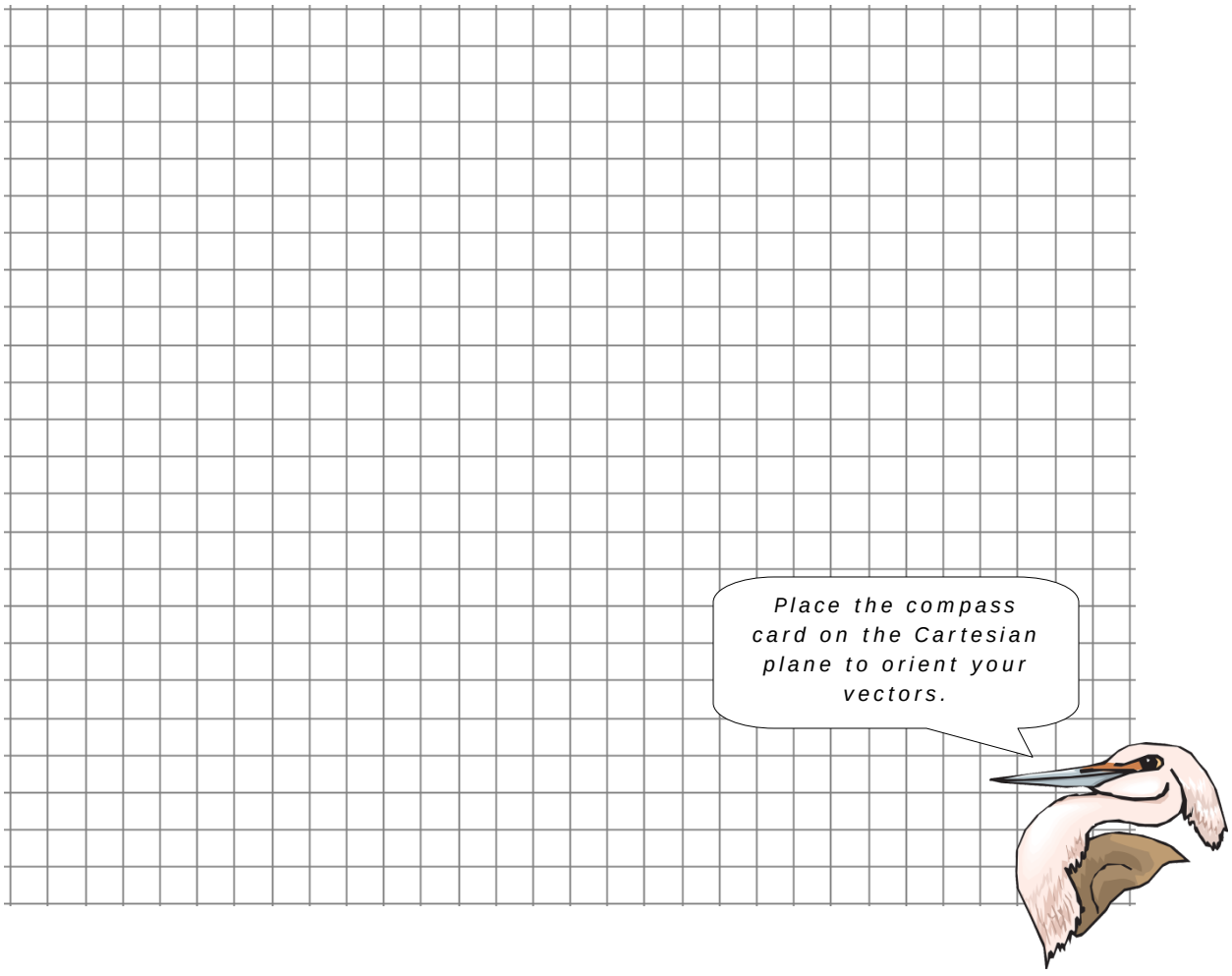
3. Given $\vec{u} = (6;3)$ and $\vec{v} = (2;5)$, algebraically calculate the components and norm of the orthogonal projection of $\vec{v}$ over $\vec{u}$. Validate with the graphical representation.



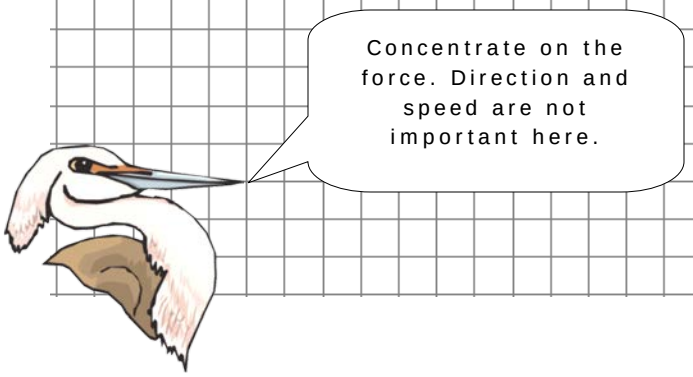
4. Vector $\vec{w}=(-4;5)$ is a linear combination of vectors $\vec{u}=(3;-2)$ and $\vec{v}=(1;4)$. Find the value of coefficients k_1 and k_2. Detail your solution.



5. A vessel navigates at a speed of 18 km/h at an angle of S 36° E. A current of 4 km/h in the direction E 10° S causes it to deviate from its course. Calculate and represent the resultant.



6. To tow a boat in difficulty, two small tugboats separated by a 35° angle must each exert a force of 9000 N. How much force would a single tugboat require to perform the same task?



Competencies evaluation

USEFUL DATA

Use the following data to solve the first three tasks.

PHYSICAL FACTORS

- The direction of the current in the seaway passage at the entrance to Lac Saint-Pierre is E 55° N and its speed is 1.2 m/s while it is E 15.8° N with a speed of 0.84 m/s at Sorel (between Sorel and St-Ignace).

EXAMPLES OF WIND CONDITIONS:

- Light wind: around 3 km/h
- Moderate wind: around 11 km/h
- Heavy wind: around 18 km/h

VESSEL DATA

Oil tanker speed: 18 km/h

Container ship speed: 16 km/h

Ferry

- Distance travelled: 1.6 km
- Crossing time: 12 minutes

Sailboat speed: 14 km/h

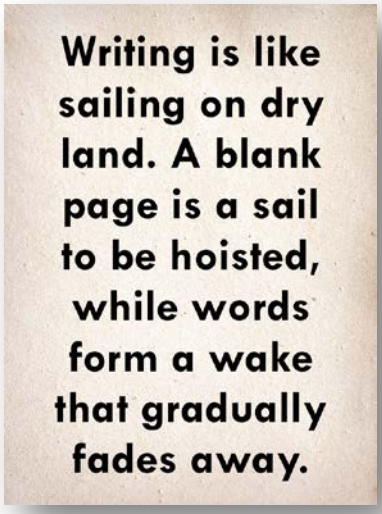
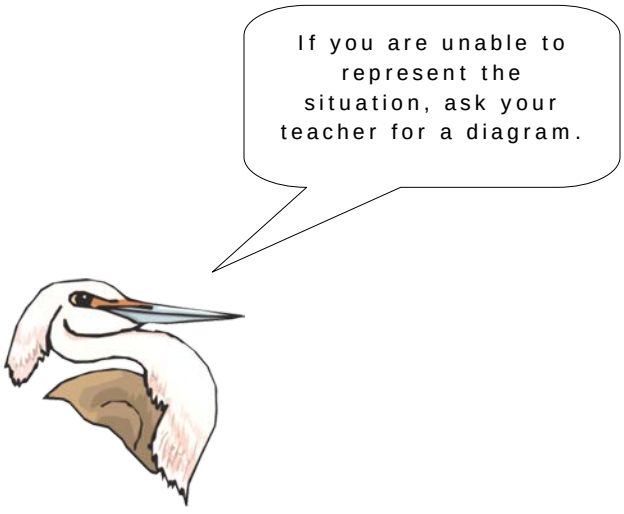


Figure 1:Erik Orsenna



TASK 1: FREIGHT TRANSPORTATION

Cargo ships carrying freight cannot cross near the Sorel islands. This is because the part of the river that is sufficiently deep for large vessels is not wide enough at this location. When two ships approach from opposite directions, which one must proceed first? The downbound oil tanker or the upbound container ship?[1] The oil tanker enters the channel at an angle of E 15.8° N (negligible wind).

Develop your arguments using vectors.

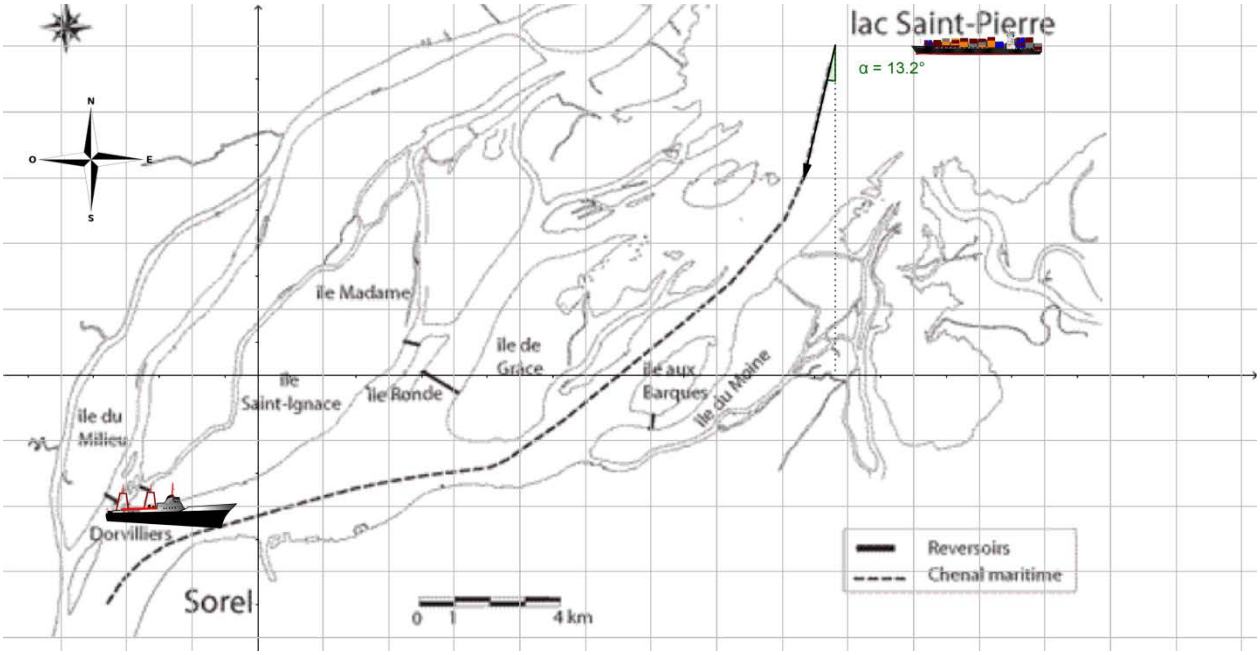
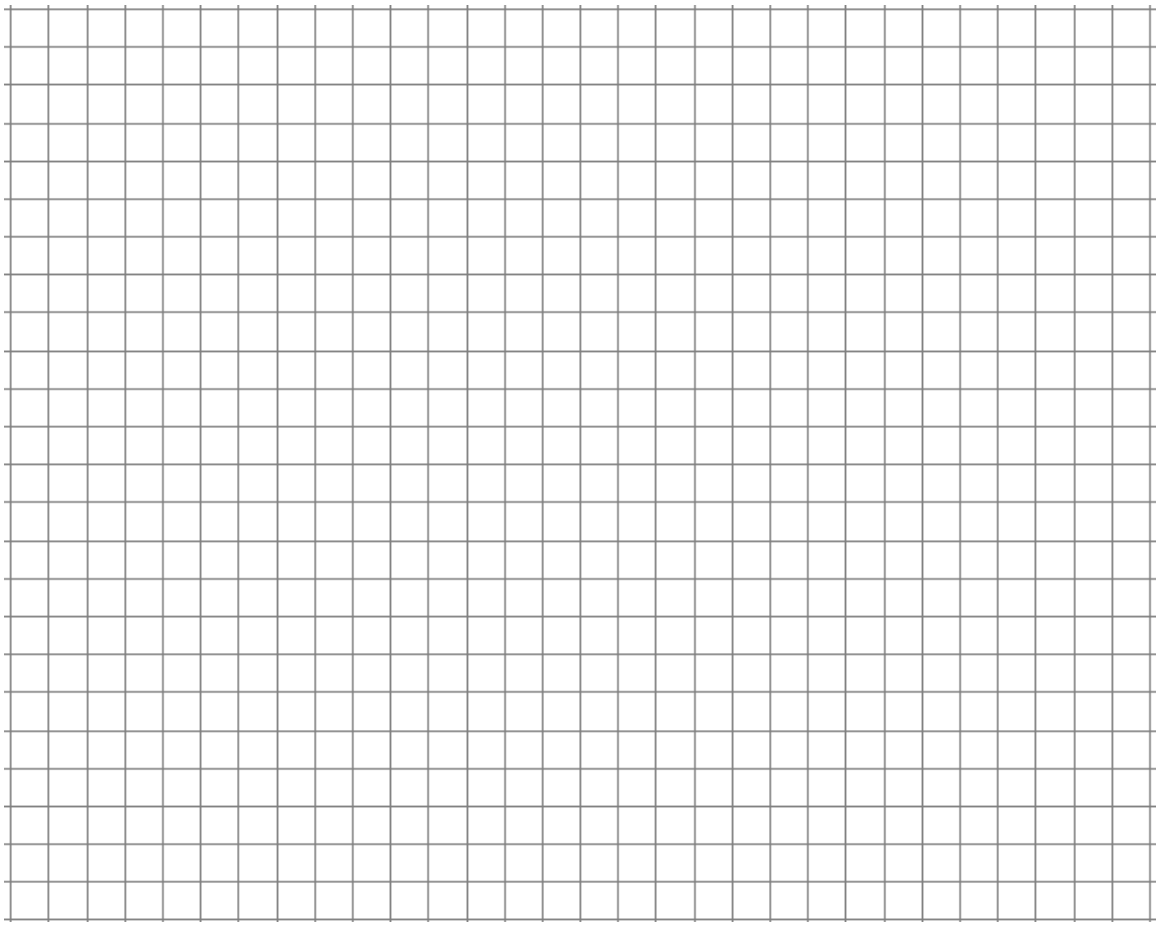


Figure 2: map: http://vertigo.revues.org/CC4.0

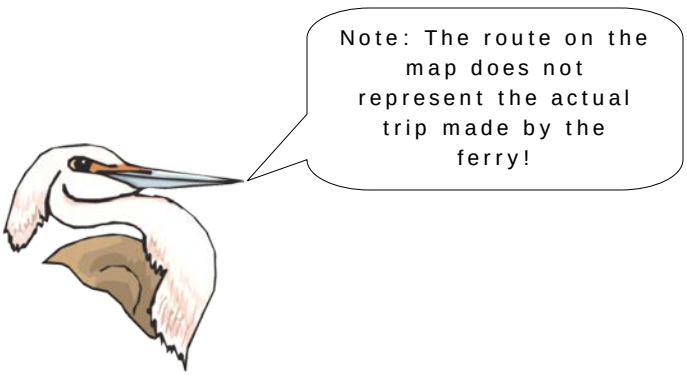
[1] In reality, the downbound ship generally has priority over the upbound ship.

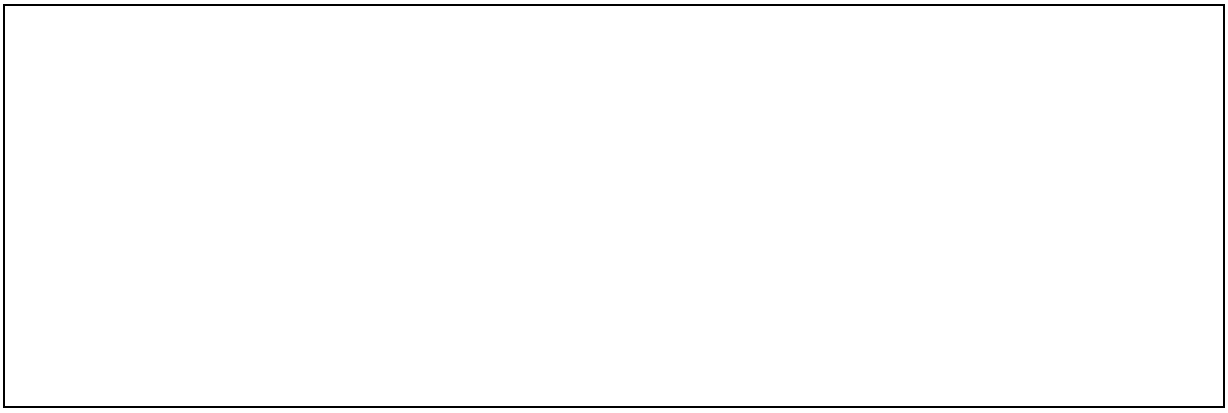


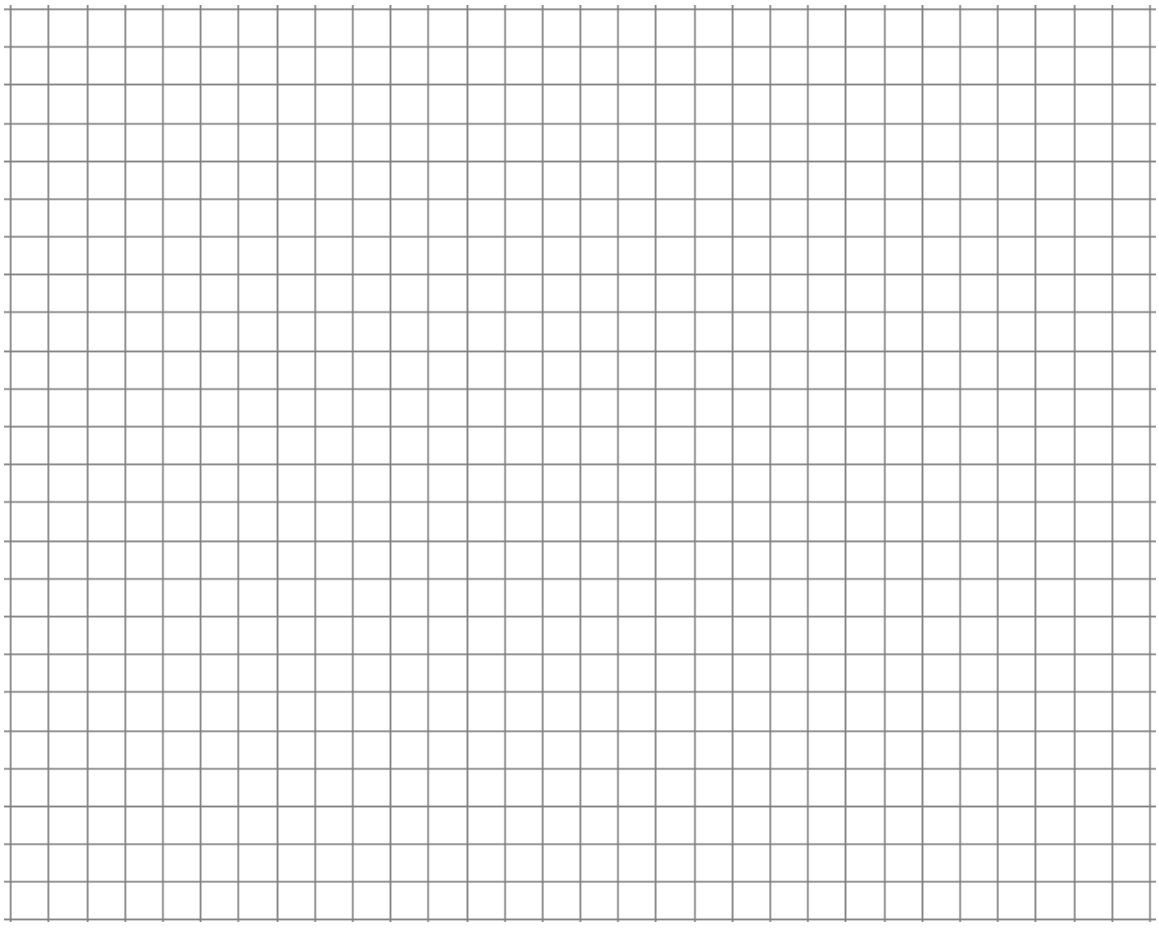
Question for reflection: Why is it easier to navigate in the same direction as the current rather than in the opposite direction?

TASK 2: BERTHIER–SOREL FERRY

Looking at the map showing the Berthier–Sorel ferry route, which direction must it head for on the outbound trip and return trip in order to reach each bank? The crossing must always be made according to the expected weather and the course may not change according to the current. (Negligible wind.)

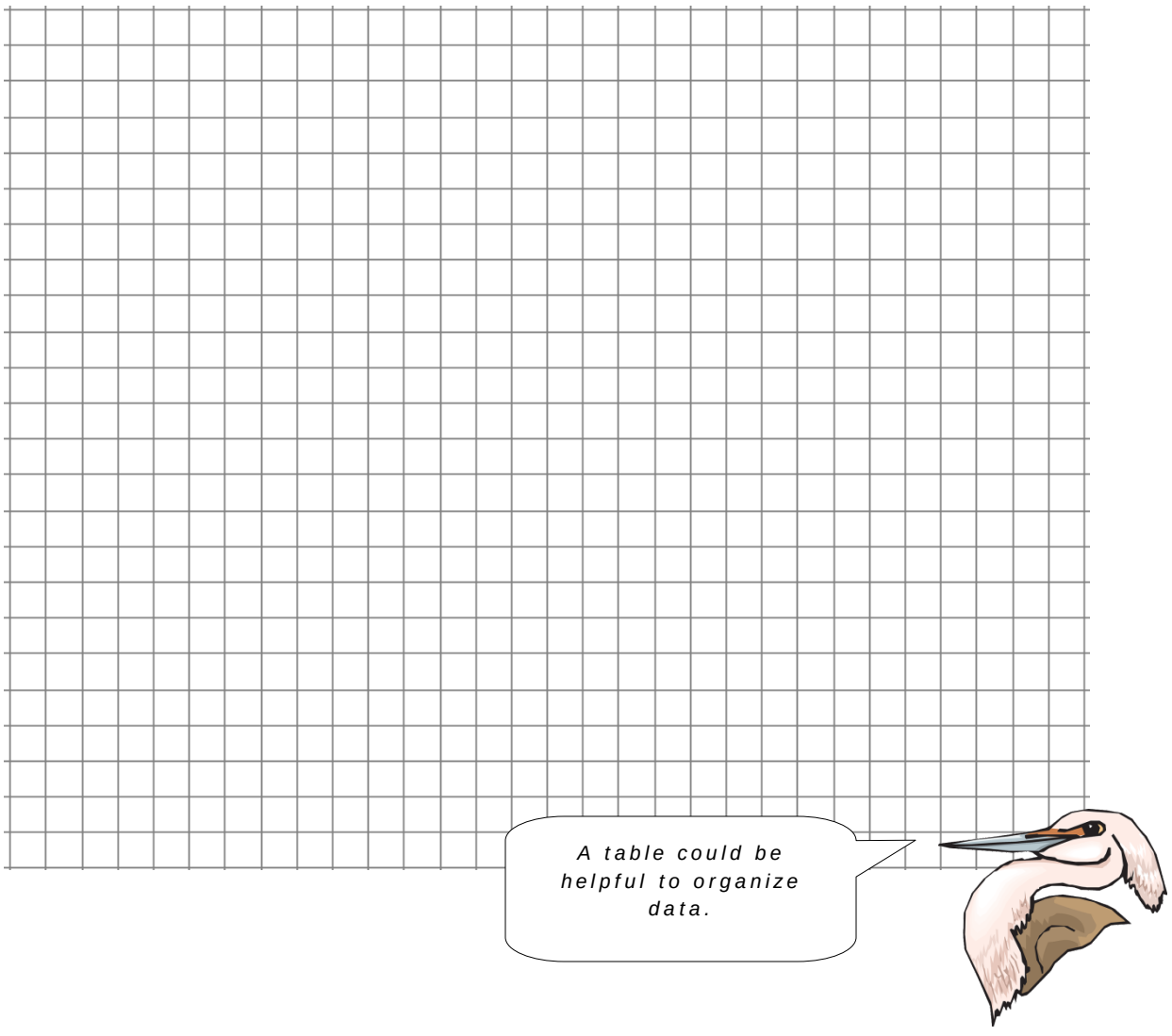




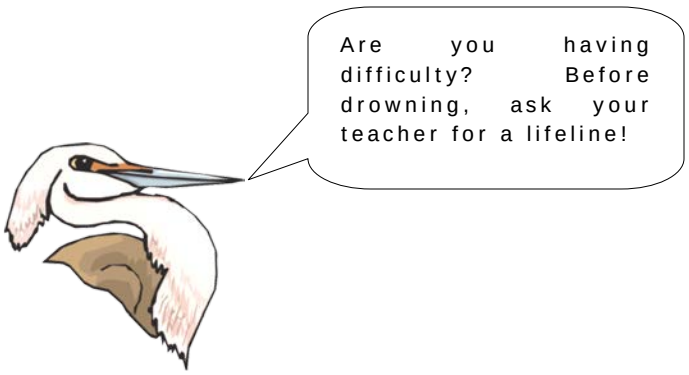


Which factor must be used to multiply the speed if the current is a) twice as fast, b) 80% of its speed?

(if necessary, use additional sheets for your calculations)



Question for reflection: How does the speed of the current affect a vessel's course?



TASK 3: SAILING CHALLENGES

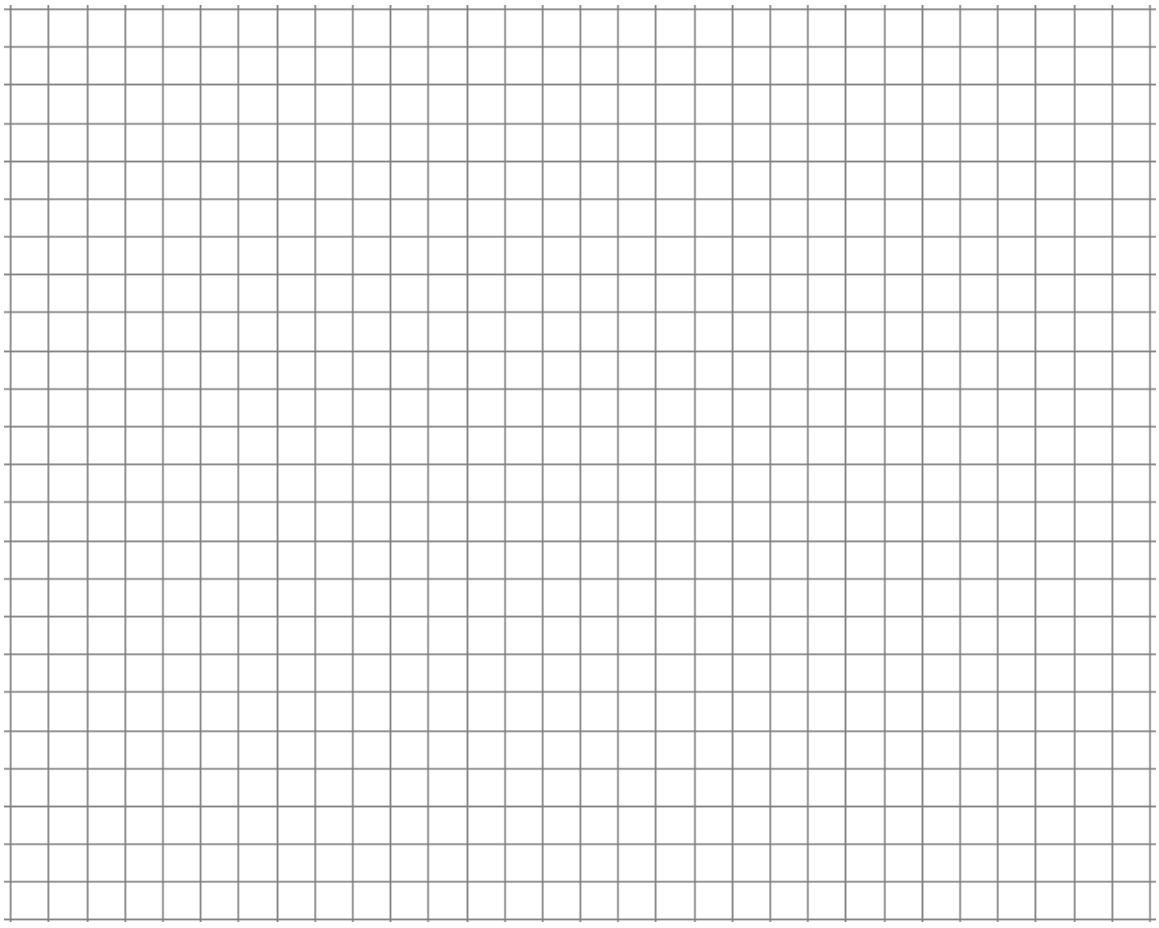
The river also attracts many sailing enthusiasts. Sailboats are propelled by the force of the wind on their sails. Is it possible for a sailboat to travel faster than the wind? Does it need to have a tail wind to do so? The higher the sailboat's speed, the higher the *apparent* wind speed. To achieve this, it is better to sail with the wind "athwartships"—i.e., with the wind coming from the side.

PART ONE

Knowing that apparent wind is the result of the true wind and the opposing movement of the sailboat (boat wind), demonstrate this statement using vectors. (Make several simulations using different wind speeds[2] and directions.) (Use technology for the simulations)

The sailboat must have a speed of 14 km/h in all cases.

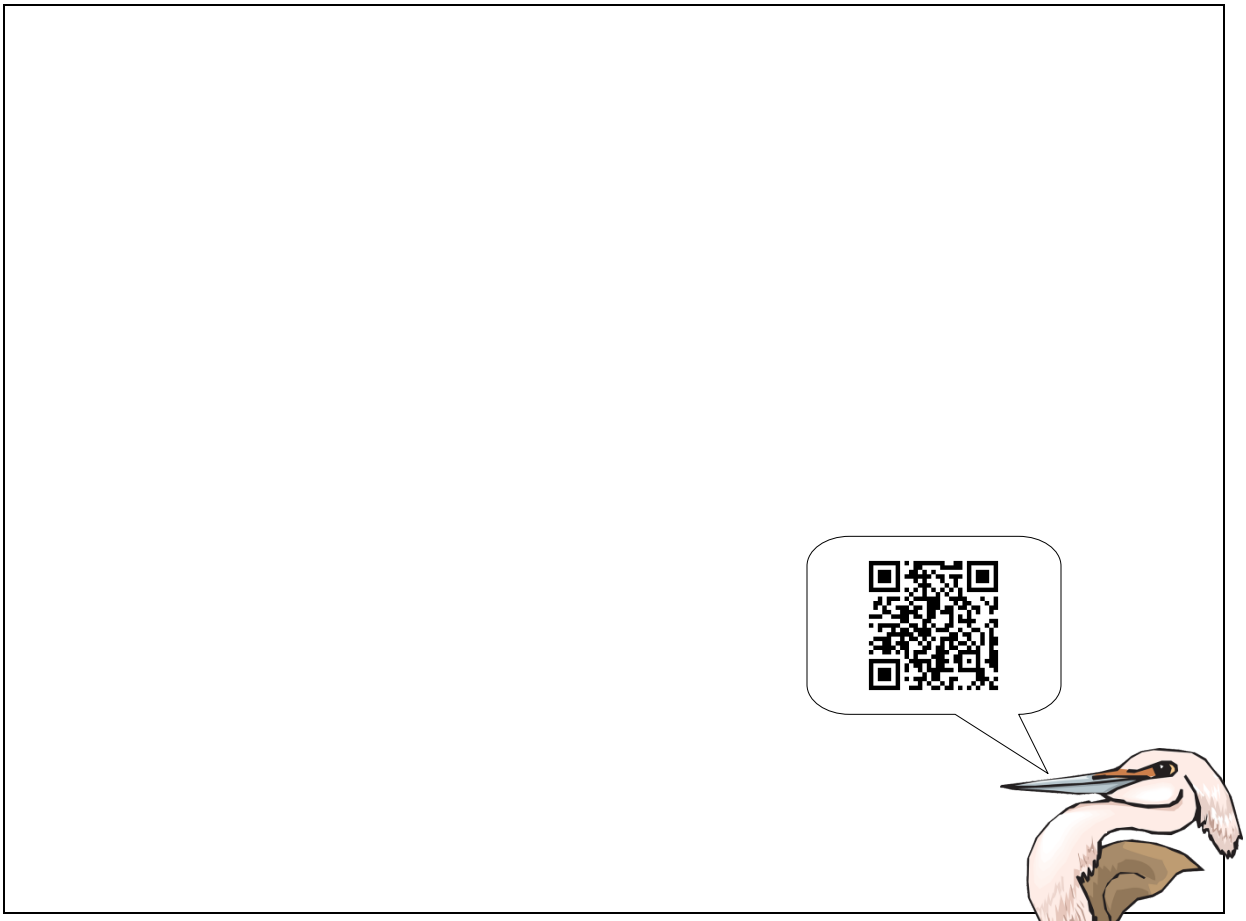
[2] Base your answers on the proposed speeds in the useful data provided at the beginning of your workbook.



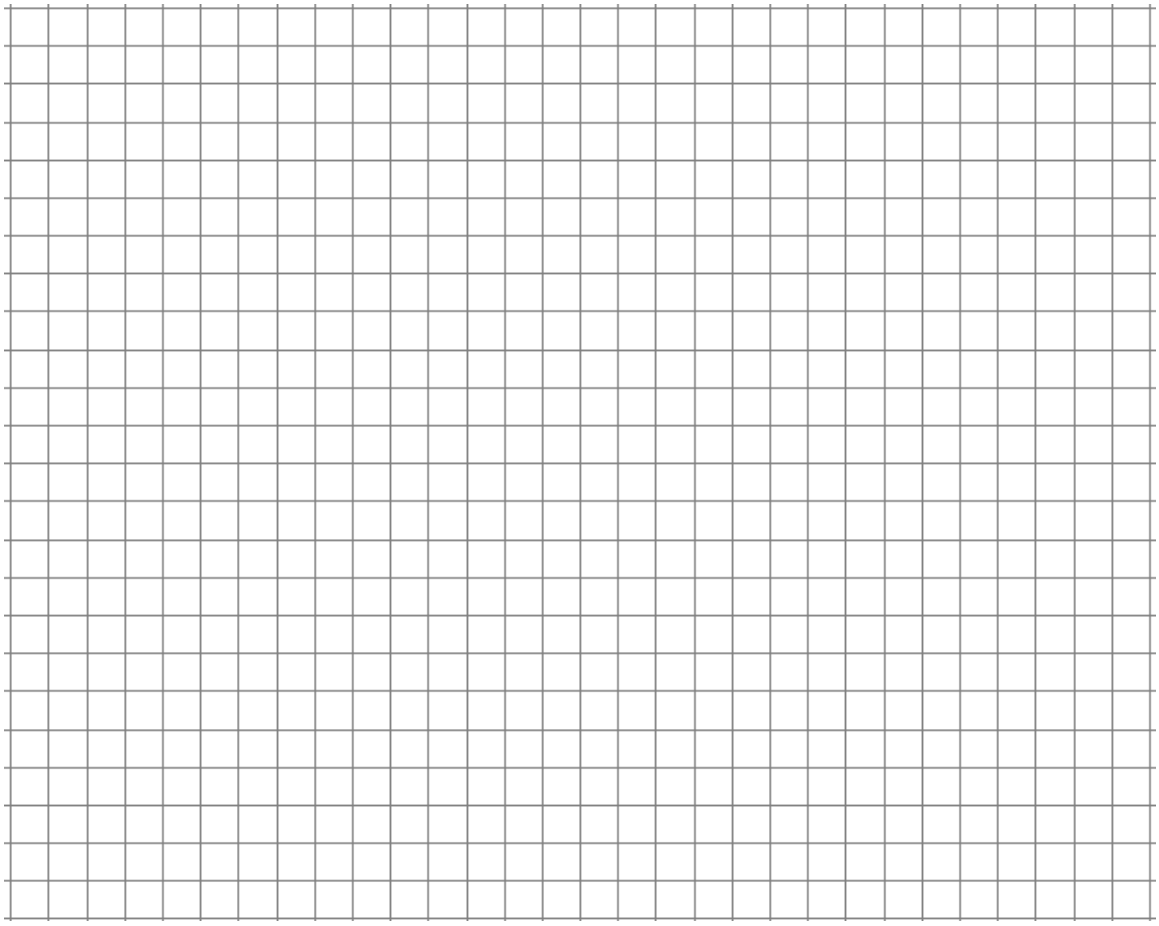
Is it possible for a sailboat to go faster than the wind? If so, under what conditions?

PART TWO

Wind exerts a force on a sail. This force is always perpendicular to the sail and is composed of propulsion force that propels the sailboat, and heeling force. Both forces are always at a right angle. The angle between the sail and the direction of the boat modifies the distribution of both types of force. What should be the angle between the direction of the sailboat and the sail if the wind force has a value of 6 and the desired propulsion force[3] has a value of 5 with a direction of E 75° N? What should the angle and heeling force be?



[3] The comparison of forces on a two-dimensional plane is analyzed in this task, hence the absence of units.



Question for reflection: In what other situation is understanding the components of force on an object essential?

TASK 4: DEMONSTRATION

Demonstrate, using three different representation methods, that when two vectors are collinear their sum is collinear for both vectors.

(if necessary, use additional sheets for your calculations)